ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics I (Math 1041)
Worksheet 1 2021/22 AY

1. For a given vectors A and B
   a) Find a real number β such that the vectors A = (β ,-3, 1) and B = (β , β , 2) are perpendicular.
   b) Find two vectors each of norm 1 that are perpendicular to the vector A = (3, 2).
   c) If U and V are perpendicular unit vectors, show that $\|U-V\| = \sqrt{2}$
   d) Vectors A and B make an angle of $\beta = \pi/6$. If $\|A\| = \sqrt{3}$, and $\|B\| = 1$.Then calculate the angle between the vectors A+B and B-A.
2. For a given vectors A and B
   a) Find the cosine of the angle between the vectors A = (4, 1, 6) and B= (-3, 0, 2).
   b) Find the projection of vector U = (-7, 1, 3) onto vector V= (5, 0, 1).
   c) Given three vectors A, B and C such that A+B+C = 0; if $\|A\|$ = 3, $\|B\|$= 1 and $\|C\|$ = 4, then evaluate A.B+B.C+A.C.
   e) Show that $(\frac{1}{2},\frac{1}{3},0),(1,1,-1)$ and $(-2,-3,5)$ are collinear points, and find symmetric equation of the line containing them.
   f) Let A be a vector in the first octant such that $\|A\| = 3$ and the direction cosines with respect to x-axis and y-axis are $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Then find vector A.
3. Show that the points $(-1,1,1),(0,2,1),(0,0,\frac{3}{2})$ and $(13,-1,5)$ lie on the same plane and find the equation of the plane containing them.
4. Find the equation of the line passing through the point (-2,5,-3) and perpendicular to the plane π : 2x – 3y + 4z + 7 = 0.
5. Find the equation of the plane:
   a) that is parallel to the z-axis and contains the points (3,-1,5) and (7,9,4).
   b) that contains the point (-2,1,4) a line L: x = 2 – 3t, y = 4 + 2t, z = 3 – 5t .
   c) that contains the point (4,0,-2) and perpendicular to each of the planes $x-y+z=0$ and $2x+y-4z=5$.
6. Let $\pi_1: \ 2x-4y+4z=7$ and $\pi_2: \ 6x+2y-3z=2$ be two planes
   a) Find parametric equations of the line of intersection of the planes π_1 and π_2.
   b) Find the cosine of the acute angle of the intersecting planes π_1 and π_2.
7. Find the point of intersection of the line L: x = 2 – 3t, y = 4 + 2t, z = 3 – 5t, t $\in \mathbb{R}$ and the plane 2x + 3y + 4z = - 8.
8. Let $L_1: \ x=1+2t, \ y=2-t, \ z=4-2t$ and $L_2: \ \frac{x-9}{1}=\frac{y-5}{3}=\frac{z+4}{-1}$ be two lines.
   a) Show that L_1 and L_2 are intersecting lines.
   b) Find the cosines of the acute angle between L_1 and L_2 at their intersection point.
   c) Find symmetric equations for the line that is perpendicular to L_1 and L_2 and passes through their intersection point.
9. Let L be the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z+5}{7}$ and π be the plane $2x+2y-z+4=0$. Find the two points on L at a distance 3 unit from π .

10. Let L_1: $\frac{x+2}{5} = \frac{y-1}{-2} = z+4$ and L_2: $\frac{x-3}{-5} = \frac{y+4}{2} = \frac{z-3}{-1}$ be two lines. Then

a) Show that L_1 and L_2 are parallel and find the distance D between L_1 and L_2.

b) Find the equation of the plane that contains L_1 and L_2.

11. Find the distance between the parallel planes: z = x + 2y +1 and 3x + 6y – 3z = 4.

12. Let $L_1:\ x = 3 - t,\ y = 4 + 4t,\ z = 1 + 2t$ and $L_2:\ x = s,\ y = 3,\ z = 2s$ be two lines

a) Show that L_1 and L_2 are skew (neither intersecting nor parallel) lines.

b) Find two planes π_1 and π_2 such that π_1 is parallel to π_2, π_1 contains L_1 and π_2 contains L_2.

c) Find the distance D of the two parallel planes π_1 and π_2 (the planes you found in (b)). (Note that this distance D is the distance between the two skew lines L_1 and L_2)

13. Let V= $\{(a, b) \mid a, b \varepsilon \Re\}$.Define addition in V and scalar multiplication by elements of ℝ as follows: (a, b) + (c, d) = (a + c, b + d) and k (a, b) = (ka, 0). Show that V is not a vector space over ℝ with respect to the operations defined. Which axiom(s) fail to hold?

14. Determine whether W is a subspace of V or not.

a) $W = \{(x, y) \in IR^2 : x - 3y = 0\},\ V = IR^2$ b) $W = \{(x, y) \in IR^2 : y = x^2\},\ V = IR^2$

c) W= $\{(a, b, c)\, \varepsilon IR^3 : a - 3b + 4c = 0\},\ V = IR^3$ d) $W = \{(x, y, z) \in IR^3 : z = 2y\},\ V = IR^3$

e) W= $\{(a, b, c) \in IR^3$: b is an integer$\}$, $V = IR^3$ f) $W = \{(a, b, c) : a \le b \le c\},\ V = IR^3$

15. Determine whether the following vectors are linearly independent or not.

a) $(1,2), (3,2)$ in IR^2 b) (-3,6), (1,-2) in IR^2 c) $(-1,2), (1,-2), (3,4)$ in IR^2

d) $(1,1,3), (0,2,1)$ in IR^3 e) $(2,1,-2), (3,2,2), (5,3,0)$ in IR^3 f) $(1,0,2), (0,-1,1), (1,3,0)$ in IR^3

16. Let $P_n = P_n(IR)$ be the vector space of all polynomials of degree less than n, $\forall n \in IN$. Determine whether the following vectors are linearly independent or not.

a) $1, x+2, 2x-1$ in P_2 b) $x-3, 1-2x$ in P_2 c) $x^2+1, x^2-x, x+1$ in P_3

d) $3-2x, x^2+2x-1, x^2+2$ in P_3 e) $1, 2x-3,\ x^2+3x-5, 2x^2+2$ in P_3

17. Let V be the set of all continuous functions on IR\

a) Show that V is a vector space under the usual addition and scalar multiplication of functions.

b) Show that the set $W = \{f \in V : f \text{ is differentiable function}\}$ is a subspace of V.

c) Show that $2^x, 2^{2x}, 2^{3x}$ are linearly independent vectors in V

d) Show that $\sin 2x, \cos 2x, \cos x \sin x$ are linearly dependent vectors in V.

18. If possible, write **u** as a linear combination of **v** and **w**.

a) $u = (1,-2),\ v = (3,1),\ w = (0.-1)$ b) $u = (-3,2),\ v = (-1,3),\ w = (2.-6)$

c) $u = (0,-2,1),\ v = (1,-1,2),\ w = (0,2,4)$ d) $u = (8,-2,1),\ v = (0,1,5),\ w = (0,3,-1)$

e) **u** = $3x^2 + 8x - 5$, **v** = $2x^2 + 3x - 4$, **w** = $x^2 - 2x - 3$